Coulomb-enhanced resonance transmission of quantum SINIS junctions

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Coherent charge transfer through a ballistic gated SINIS (here S stands for a superconductor, N is a normal-metal island, and I is an insulator) junction is mediated by the resonant tunneling via the Andreev states. Extra charge accommodated on the Andreev levels partially compensates the charge induced by the gate voltage preserving the electron wavelength and maintaining the resonance conditions in a broad range of gate voltages. As a result, the transparency of the junction as well as the supercurrent though it can be substantially increased as compared to the zero-Coulomb case.

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I. INTRODUCTION

According to the classical picture of the Coulomb blockade in gated structures[1] the Coulomb interaction usually suppresses electronic transport through small conductors by introducing an additional energy barrier associated with the charging energy. In this Brief Report we show that in ballistic SINIS junctions (here S stands for a superconductor, N is a normal-metal island, and I is an insulator), the Coulomb interaction can rather *stimulate* the supercurrent than suppress it. The supercurrent through a junction containing a ballistic normal island is mediated by Andreev states with energies controlled by the effective transparency of the double-barrier structure which is determined by the wavelength of electrons. The extra charge on the dot affects not only the energy but also the wavelength of electrons. This makes the transmission extremely sensitive both to the gate voltage and to the charge on the Andreev states localized in the normal island. We show that charging of Andreev states can actually *preserve* the condition of resonance tunneling: the charge adjusts itself so as to compensate the deviation of the chemical potential caused by the change in the gate voltage thus increasing the transparency of the device for the supercurrent.

II. SYSTEM

We consider the normal island in the form of a short single-mode ballistic conductor (wire) connected to bulk superconducting leads via low-transparency contacts. This corresponds to the common experimental situation which is realized, in particular, in recent experiments on nanowires.[2] The length d of a conductor is much less than the superconducting coherence length, ξ, i.e., $|\Delta| \ll \delta E$ where $\delta E \sim \hbar v_F/d$ is the level spacing in the double-barrier structure, v_F is the velocity of electrons in a quantum conductor, and $|\Delta|$ is the superconducting gap in the leads. The short conductor (sometimes called an Andreev quantum dot) is weakly coupled to the superconducting leads and forms a low-proximity device. This setup is similar to that considered in Ref. 3 where the charging of Andreev levels was studied in the limit of small Coulomb interaction.

To characterize the efficiency of the Coulomb interaction in the SINIS we introduce the dimensionless parameter $\gamma = E_C/\delta E \sim e^2 d/C\hbar v_F$, where $E_C = e^2/2C$ is the Coulomb energy of the island, C being its capacitance. The maximum value of γ is less than $e^2/\hbar v_F \varkappa$ with the island capacitance $C \sim \varkappa d \ln(d/a)$, where a is the transverse dimension of the wire and $\varkappa$ is the effective dielectric constant in the presence of a substrate. For a typical value $v_F \sim 10^8$ cm/s the corresponding γ is small; thus the assumption $\gamma \ll 1$ is adequate for practical devices. The same parameter characterizes the Luttinger-liquid effects,[4] which thus can be neglected. However, in spite of the small value of γ, the Coulomb energy can still be either larger or smaller than the superconducting gap. To account for nonlinear backaction effects on the contact transparency associated with charging of the Andreev level in case of a considerable Coulomb energy $E_C \sim |\Delta|$, we adopt a mean-field model similar to that employed in Ref. 5 for normal junctions. In this respect, our model differs from those considering transport through quantum dots having a single *fixed* electron level.[6,7]

III. MODEL

The charging Hamiltonian has the form

$$\hat{H}_c = E_C(\hat{N} - N_0)^2, \quad \hat{N} = \sum_\alpha \int_V \hat{\psi}_\alpha^\dagger(\mathbf{r})\hat{\psi}_\alpha(\mathbf{r})d^3r \qquad (1)$$

the sum being taken over the spin indices α and the integration domain is the volume V of the island. Here N_0 is the background charge supplied by ions and by the external circuit. The kinetic energy has the usual form

$$H_{\rm kin} = \sum_\alpha \int d^3r \hat{\psi}_\alpha^\dagger(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} - E_F + V(\mathbf{r})\right]\hat{\psi}_\alpha(\mathbf{r}),$$

where $V(\mathbf{r}) = I[\delta(x+d/2) + \delta(x-d/2)]$ describes the insulating barrier between the island and the leads, x is the coordinate along the conductor.

Superconductivity in the leads is described by the usual BCS Hamiltonian where $\Delta(\mathbf{r})=|\Delta|e^{i\chi_{L,R}}$ inside the leads, i.e., for $x<-d/2$ or $x>d/2$, respectively, while $\Delta=0$ in the normal conductor, $-d/2<x<d/2$. In this model, the states of the system are spin independent. The advanced and retarded Green's functions in the real-frequency representation can be found from the Dyson equation

$$\int d^3r \check{L}(\mathbf{r}_1,\mathbf{r})\check{G}^{R(A)}_{\epsilon}(\mathbf{r},\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{2}$$

where

$$\check{L}(\mathbf{r}_1,\mathbf{r})=\check{G}_0^{-1}(\mathbf{r}_1)\delta(\mathbf{r}-\mathbf{r}_1)+2E_C h(\mathbf{r}_1)\check{K}(\mathbf{r}_1,\mathbf{r}),$$

$$\check{G}_0^{-1}(\epsilon,\mathbf{r})=\begin{pmatrix}-\epsilon+\hat{H}_0 & -\Delta\\ \Delta^* & \epsilon+\hat{H}_0\end{pmatrix},\quad \check{G}=\begin{pmatrix}G & F\\ -F^\dagger & \bar{G}\end{pmatrix}$$

are matrices in the Nambu space and

$$\hat{H}_0=-\hbar^2\nabla^2/2m-E_F+V(\mathbf{r})+U_C h(\mathbf{r}),$$

$$U_C=2E_C(N-N_0),\quad N=-\int_V d^3r\ \mathrm{Tr}\check{K}(\mathbf{r},\mathbf{r}).$$

The function $h(\mathbf{r})=1$ if $\mathbf{r}$ belongs to the island and $h(\mathbf{r})=0$ otherwise. The kernel

$$\check{K}(\mathbf{r}_1,\mathbf{r})=\int\frac{d\epsilon}{4\pi i}\check{G}^K_{\epsilon}(\mathbf{r}_1,\mathbf{r}) \tag{3}$$

is determined by the Keldysh function, $\check{G}^K_{\epsilon}(\mathbf{r}_1,\mathbf{r}_2)=[\check{G}^R_{\epsilon}(\mathbf{r}_1,\mathbf{r}_2)-\check{G}^A_{\epsilon}(\mathbf{r}_1,\mathbf{r}_2)]f_1(\epsilon)$. Here $f_1(\epsilon)=1-2n_\epsilon$ and n_ϵ is the occupation number. In equilibrium n_ϵ is the Fermi function and $f_1(\epsilon)=\tanh(\epsilon/2T)$. The energy U_C describes the variation in the charge on the island due to the change in the number of excitations while N has a meaning of the excitation charge on the island.

IV. BOGOLIUBOV-DE GENNES (BdG) EQUATIONS

Let us expand the retarded and advanced Green's functions in a set of orthogonal and normalized functions as

$$\check{G}^{R(A)}_{\epsilon}(\mathbf{r}_1,\mathbf{r}_2)=\sum_n\frac{\check{U}_n(\mathbf{r}_1)\check{U}_n^\dagger(\mathbf{r}_2)}{E_n-\epsilon\mp i\delta},\quad \check{U}_n(\mathbf{r})=\begin{pmatrix}u_n(\mathbf{r})\\ -v_n(\mathbf{r})\end{pmatrix}$$

$\check{U}_n^\dagger=(u_n^*,v_n^*)$. The functions u and v satisfy the linear equations with effective potentials determined by $K_{uu}(\mathbf{r},\mathbf{r}_1)$ and $K_{uv}(\mathbf{r},\mathbf{r}_1)$

$$\hat{H}_0u_n(\mathbf{r})+\Delta(\mathbf{r})v_n(\mathbf{r})+E_C h(\mathbf{r})\int_V K_{uu}(\mathbf{r},\mathbf{r}_1)u_n(\mathbf{r}_1)d^3r_1$$

$$-E_C h(\mathbf{r})\int_V K_{uv}(\mathbf{r},\mathbf{r}_1)v_n(\mathbf{r}_1)d^3r_1=E_n u_n(\mathbf{r}), \tag{4}$$

$$\hat{H}_0v_n(\mathbf{r})-\Delta^*(\mathbf{r})u_n(\mathbf{r})+E_C h(\mathbf{r})\int_V K^*_{uu}(\mathbf{r},\mathbf{r}_1)v_n(\mathbf{r}_1)d^3r_1$$

$$+E_C h(\mathbf{r})\int_V K^*_{uv}(\mathbf{r},\mathbf{r}_1)u_n(\mathbf{r}_1)d^3r_1=-E_n v_n(\mathbf{r}). \tag{5}$$

The kernels $K_{uu}(\mathbf{r},\mathbf{r}_1)$ and $K_{uv}(\mathbf{r},\mathbf{r}_1)$ are determined self-consistently. They can also be expanded into the same wave functions

$$K_{uu}(\mathbf{r},\mathbf{r}_1)=\sum_m u_m(\mathbf{r})u_m^*(\mathbf{r}_1)f_1(E_m),$$

$$K_{uv}(\mathbf{r},\mathbf{r}_1)=\sum_m u_m(\mathbf{r})v_m^*(\mathbf{r}_1)f_1(E_m). \tag{6}$$

The kernels obey the symmetry $K_{uu}(\mathbf{r},\mathbf{r}_1)=-K^*_{vv}(\mathbf{r},\mathbf{r}_1)=K^*_{uu}(\mathbf{r}_1,\mathbf{r})$ and $K_{uv}(\mathbf{r},\mathbf{r}_1)=K^*_{vu}(\mathbf{r},\mathbf{r}_1)=K_{uv}(\mathbf{r}_1,\mathbf{r})$ due to the symmetry of the functions $u_E\to v^*_{-E}, v_E\to -u^*_{-E}$ with respect to $E\to -E$. The excitation charge is also expressed through u_n and v_n, $N=\frac{1}{2}\Sigma_n N_n$, where

$$N_n=-\int dV[u_n^*(\mathbf{r})u_n(\mathbf{r})-v_n^*(\mathbf{r})v_n(\mathbf{r})]f_1(E_n) \tag{7}$$

is the number of excitations on the state n, it determines the average excitation charge $Q_n=eN_n$ carried by the state. The sum runs over the states with both positive and negative energies. It can be written as a sum over only positive-energy states: $N=\Sigma_{n,E_n>0}N_n$.

V. ANDREEV STATES

The normal conductor has a single-transport mode parameterized by the coordinate x along it. The eigenstates in a short low-transparency junction have the form of narrow particlelike and holelike resonances such that only two of them fit into the subgap range of $|\Delta|$. Due to proximity to the superconducting leads these states transform into the Andreev states localized over the distances on the order of ξ near the junction. All other resonances are essentially the same as in the normal state; thus they can be ignored if one is interested in energies on the order of the superconducting gap. The two Andreev states will be labeled with $n=+1$ and $n=-1$. One of them has a positive and other has a negative energy $E_1=-E_{-1}>0$. In what follows we assume that the transparency is so small that $\mathcal{T}\ll d/\xi$. In this limit the Andreev states are mostly concentrated inside the normal conductor. We will also consider energies much smaller than the superconducting gap because it is this energy range where the supercurrent enhancement is most pronounced. In this case the kernels, Eqs. (6), in Eqs. (4) and (5) contain contributions only from the two Andreev states, $n=\pm 1$. Due to the symmetry with respect to $E\to -E$

$$K_{uu}(x,x_1)=[u_1(x)u_1^*(x_1)-v_1^*(x)v_1(x_1)]f_1(E_1),$$

$$K_{uv}(x,x_1)=[u_1(x)v_1^*(x_1)+v_1^*(x)u_1(x_1)]f_1(E_1).$$

Denoting $u_1=u^+e^{ip_{u}x}+u^-e^{-ip_{u}x}$ and $v_1=v^+e^{ip_{v}x}+v^-e^{-ip_{v}x}$ and neglecting the difference between the momenta of u and v in

the kernels, $p_u=p_v\equiv p$ (for a short junction, $p_u-p_v\sim E/v_F \ll 1/d$) we find

$$\int_{-d/2}^{d/2} dx_1[K_{uu}(x,x_1)u_1(x_1)-K_{uv}(x,x_1)v_1(x_1)]=-u_1(x)N_1 - v_1^*(x)M_1,$$

$$\int_{-d/2}^{d/2} dx_1[K_{uu}^*(x,x_1)v_1(x_1)+K_{uv}^*(x,x_1)u_1(x_1)]=-v_1(x)N_1 + u_1^*(x)M_1$$

N_1 being the number of excitations on the state $n=1$ while $M_1=2\int_{-d/2}^{d/2}u_1(x_1)v_1(x_1)f_1(E_1)dx_1$.

The BdG Eqs. (4) and (5) inside the normal conductor take the form

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-\mu\right)u_1-E_CM_1v_1^*=E_1u_1,$$

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-\mu\right)v_1+E_CM_1u_1^*=-E_1v_1,$$

where the effective chemical potential is

$$\mu=E_F-U_C+E_CN_1. \tag{8}$$

For $u,v^*\propto e^{ipx}$ we find from here

$$p^2/2m-\mu=\pm\sqrt{E_1^2-E_g^2}, \tag{9}$$

where $E_g=E_C|M_1|$ is the Coulomb gap. Using the solutions of the BdG equations without the Coulomb interaction, one can show that $M_1\sim\mathcal{T}$. The Andreev state energy is (see below) $E_1\gtrsim(\hbar v_F/d)\mathcal{T}$. Therefore, the Coulomb gap $E_g\sim E_C\mathcal{T}$ in Eq. (9) can be neglected when $\gamma\ll 1$. In this limit Eq. (9) reduces to its usual form, $p^2/2m-\mu=\pm E_1$ while the functions u and v inside the normal conductor are decoupled.

The BdG equations for the entire system can thus be written in a standard form

$$[\hat{H}_0-h(\mathbf{r})E_CN_1]u_1(\mathbf{r})+\Delta(\mathbf{r})v_1(\mathbf{r})=E_1u_1(\mathbf{r}), \tag{10}$$

$$-[\hat{H}_0-h(\mathbf{r})E_CN_1]v_1(\mathbf{r})+\Delta^*(\mathbf{r})u_1(\mathbf{r})=E_1v_1(\mathbf{r}). \tag{11}$$

The BdG Eqs. (10) and (11) determine the energy of the Andreev bound states as a function of the phase difference $\phi=\chi_R-\chi_L$. For low transparency, $\mathcal{T}\ll d/\xi$, the energies $E_{\pm 1}=\pm(\hbar v_F/d)\epsilon_A$ in the range $E\ll|\Delta|$ have the form[3,8]

$$\epsilon_A=\sqrt{z^2+\mathcal{T}^2\cos^2(\phi/2)}, \tag{12}$$

where $\mathcal{T}=\mathcal{T}_1/2$ is the transparency of two contacts in series. The quantity z measures the shift of the chemical potential μ inside the conductor, Eq. (8), from one of the resonances. It is defined through the equation

$$p_Fd/\hbar+\eta+(d/\hbar v_F)[E_CN_1-U_C]=\pi k+z, \tag{13}$$

where $\eta=\arctan\sqrt{\mathcal{T}_1}$ is the scattering phase shift and k is an integer. The range of applicability of Eq. (12) is $\epsilon_A\ll d/\xi$.

As it follows from Eq. (7), $U_C=U_C'+2E_CN_1$, where $U_C'=2E_C(N'-N_0)$ and $N'=\Sigma_{n\neq 1}N_n$. Let us denote

$$y=p_Fd/\hbar+\eta-dU_C'/\hbar v_F-\pi k.$$

The quantity y depends on the gate voltage through the background charge N_0. The excitation number N' changes little if the chemical potential varies within one resonance, $z\lesssim 1$. Therefore, the chemical potential shift y can be considered as a parameter controlled by the gate voltage.

For a localized state[8]

$$N_1=\left(\frac{\partial E_1}{\partial\mu}\right)_\phi f_1(E_1). \tag{14}$$

The subscript ϕ means that the phase is kept constant. As it follows from Eq. (8) for the effective chemical potential of the Andreev state, $\partial\mu/\partial z=\hbar v_F/d$. Therefore, Eq. (13) yields

$$y=z+\gamma\frac{\partial\epsilon_A}{\partial z}f_1(E_1). \tag{15}$$

We consider zero temperatures and put $f_1(E_1)=1$. This can be done if temperature satisfies $T\ll\mathcal{T}\delta E\sim\Delta\mathcal{T}(\xi/d)$. Equation (15) is the central result of this Brief Report. Equations (12) and (15) determine ϵ_A as a function of y and Eq. (14) then determines the charge N_1 on the Andreev level.

VI. RESULTS

Equations (12) and (15) yield

$$y=(1+\gamma/\epsilon_A)\sqrt{\epsilon_A^2-\mathcal{T}^2\cos^2(\phi/2)}. \tag{16}$$

We see that at $\gamma\neq 0$ (i.e., due to the Coulomb interaction) there appears a substantial region of gate voltages, $y\lesssim\gamma$, where energy is close to its smallest possible value, $\mathcal{T}\cos(\phi/2)$. This is more pronounced if the Coulomb energy is larger than the gap $E_C/|\Delta|\gg 1$ (estimates will be given later). Indeed, since $\epsilon_A\lesssim d/\xi$ the ratio $\gamma/\epsilon_A>E_C/|\Delta|$. Yet more drastic effect is expected if $\mathcal{T}\ll\gamma$. In this case the term γ/ϵ_A in brackets in Eq. (16) dominates. As a result, for $y<\gamma$

$$\epsilon_A=\frac{\mathcal{T}\cos(\phi/2)}{\sqrt{1-y^2/\gamma^2}}. \tag{17}$$

According to Eq. (16) and (17), ϵ_A and z remain small $\epsilon_A,z\ll\gamma$ in the range of gate voltages $y\lesssim\gamma$. Thus the energy satisfies $E_1\ll E_C,|\Delta|$ and the Kondo regime is not realized.

Figure 1 (upper panels) shows the solutions of Eqs. (16) for $\mathcal{T}=0.1$ and for two values of the Coulomb interaction, $\gamma=0.2$ (left panel) and $\gamma=0$ (right panel). One observes that $\epsilon_A\to 0$ as ϕ approaches π if $y<\gamma$. The decrease in energy is, of course, cut off when $|\phi-\pi|\sim(T/|\Delta|)(d/\xi\mathcal{T})$ because of the decreasing factor $f_1(E_1)$ in Eq. (15). This energy behavior has a crucial impact on the supercurrent given by the expression[9]

$$j_s=-\frac{2e}{\hbar}\left(\frac{\partial E_1}{\partial\phi}\right)_\mu f_1(E_1)=j_0\frac{\mathcal{T}\sin\phi}{\epsilon_A}, \tag{18}$$

where $j_0=ev_F\mathcal{T}/2d$. Since the energy remains small in a wide range of gate voltages, the supercurrent is essentially enhanced as compared to its zero-Coulomb value for the same

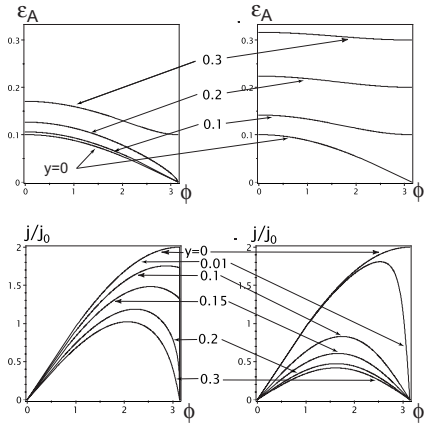

FIG. 1. Andreev state energy (upper panels) and the supercurrent (lower panels) in the presence of Coulomb interaction, $\gamma=0.2$ (left panels) and without it, $\gamma=0$ (right panels) as functions of the phase difference ϕ and the gate voltage y for $\mathcal{T}=0.1$. The curves for $\gamma=0.2$ and $\gamma=0$ coincide at $y\to 0$. The curves for the supercurrent at $\gamma=0.2$ for $y=0$ and $y=0.01$ overlap while at $\gamma=0$ they significantly differ at ϕ close to π.

gate voltage. In particular, for comparatively large Coulomb interaction, $\gamma \gg \mathcal{T}$, we have from Eq. (17)

$$j/j_0 = 2\sqrt{1-y^2/\gamma^2}\,\sin(\phi/2).$$

The critical current $j_c=2j_0\sqrt{1-y^2/\gamma^2}$ is reached at $\phi=\pi$. It is roughly by factor $\gamma/\mathcal{T}$ larger than its value without the Coulomb interaction, $j_c=j_0\mathcal{T}/y$, reached at $\phi=\pi/2$ for the same gate voltage $y \gg \mathcal{T}$. The supercurrent Eq. (18) is shown in Fig. 1 (lower panels) for $\mathcal{T}=0.1$ and for two value of the Coulomb interaction, $\gamma=0.2$ (left panel) and $\gamma=0$ (right panel). One observes significant enhancement of the supercurrent for a given gate voltage.

VII. DISCUSSION

We have shown that charging of the Andreev level leads to a strong enhancement of the supercurrent in a wide range of gate voltages (parameterized by the quantity y). The charge in the Andreev state compensates the deviation of the chemical potential caused by the change in the gate voltage thus preserving the resonance transmission of the double-barrier structure and increasing the transparency of the device. A tendency to current enhancement in such systems was already noticed in Ref. 7 in the limit of small Coulomb interaction.

The Coulomb interaction is characterized by the dimensionless parameter $\gamma=dE_C/\hbar v_F$. The upper estimate gives $\gamma \sim e^2/\hbar v_F \lesssim 1$. Our approximation requires small γ. Nevertheless, the factor $\gamma/\mathcal{T}$ that determines the enhancement of the supercurrent can reach quite large values due to low transparency. A substantial enhancement of supercurrent occurs already when $E_C \gtrsim |\Delta|$. A typical gap Δ (as for Al) is about 1 K. The Coulomb energy $E_C=e^2/2C$ of 1 K corresponds to the capacitance $C \approx 8\times 10^{-4}$ cm. Therefore, to have $E_C \gg \Delta$ the characteristic size of the normal conductor should be smaller than $\sim 10^{-3}$ cm which can be easily realized in practice.

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